ISSN 2319–7757 EISSN 2319–7765

Indian Journal of Engineering

Dynamic Modelling of a Two Wheeled Mobile Manipulator

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Publication History
Received: 14 October 2016
Accepted: 09 November 2016
Published: January-March 2017

Citation
Animesh Chhotray, Priyadarshi Biplab Kumar, Dayal R Parhi. Dynamic Modelling of a Two Wheeled Mobile Manipulator. *Indian Journal of Engineering*, 2017, 14(35), 14-19

ABSTRACT

Robotic Mobility in dynamic environments has gained popularity among many researchers over past few years. The mechanical stability criteria of intelligent robots like two wheeled self-balancing mobile manipulators have become an important subject of research in recent time. The balancing on two wheels and spin on the spot ability with smaller footprints of the two wheeled mobile robots distinguish them from other multi-wheel counterparts. It is highly essential for stabilizing the robot's body posture and trajectory control during navigation in a clumsy environment. Hence the present work analyses the exact dynamics of the mechanisms involved, using governing equations of motion for further design and implementation. The necessary sensory informations about the tilt angle are obtained from both accelerometer and gyroscope collectively. Finally the dynamic system modeling is mathematically established through both DC motor electrical model and two wheeled inverted cart model.

Key Words: Two Wheeled Mobile Robot, Manipulator, Self-Balancing Criteria, Dynamic Model.

1. INTRODUCTION

The invention of machine tools in nineteenth century and subsequent development of computers in twentieth century provide the technology to build facsimiles of the brain and body parts of different creatures in the form of robots. Basically, these robots are initially employed at monotonous and repetitive works inside industries to accelerate productivity. The first articulated robot concept was brought forward by George Devol in 1960. Initially the industrial robots or else named as manipulators are having fixed base and are taught to perform some sequential tasks in a confined workspace. In due time fusion of sensors facilitate independent data acquisition from the environment and subsequent implementation of different artificial techniques make them more autonomous in nature.

Now a days due to technological advancement the use of robots are not confined only in industrial organizations; rather their involvement spread out to many domestic purpose fulfillment like assisting human beings in their normal routine activities in both home and offices. For performing such kind of task the robots must be quite intelligent and autonomous in nature so that it can work independently in an environment quite similar to practical situations like flat surfaces, slopes, sharp turns, steps as well as narrow spaces. Hence, the present work describes about a two wheeled mobile manipulator, which is a combined system of an inverted pendulum, manipulator, and mobile platform. The advantage of this fusion lie upon the dynamic stability feature of the inverted pendulum, which actively stabilizes itself even the system is quite tall. Unlike conventional manipulators engaged in industries with fixed base are having limited workspace but by combining this wheeled mobile platform with manipulators the work space increases. Therefore, the two-wheeled mobile manipulators (as in figure 1,2) show an excellent maneuverability in confined space with smaller footprints and more flexibility than statically stable multi wheel robots. The inherent dynamic stability concept is described mathematically by considering both DC motor electrical model and inverted cart model.

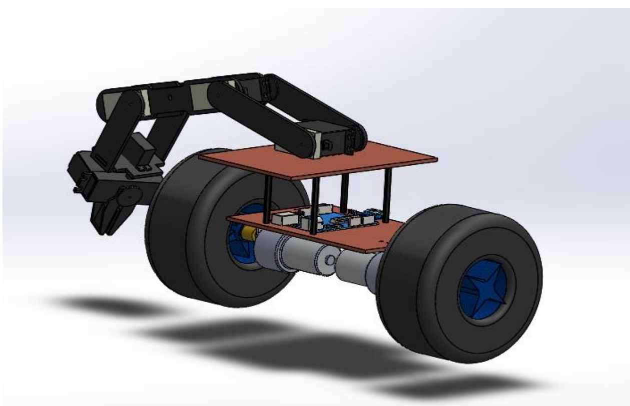

Figure 1 Solid Modelling of Two Wheeled Mobile Manipulator

Figure 2 Lab Built Two Wheeled Mobile Manipulator

Deegan et al. [1] assessed the UMASS uBot-4, which is a dynamically stable two-wheeled mobile manipulator, and they motivated for its advanced design. This series of the robot have the platform for whole body postural control relevant to the force to be render at manipulator end-effecter. Ren et al. [2] developed a dynamical model for the motion of two-wheeled Vehicle which is intrinsically unstable. Due to the nonlinearity of mathematical model Neural-Network like self tunning PID control is proposed for controlling parameters automatically.. Abeygunawardhana et al. [3] developed a real-time Gain Control Technique for improving the stability of two-wheel mobile manipulator. The kinematic and dynamic model for a coaxial two-wheeled system mounted with three-link manipulator was established. The proposed Control Technique improved the performance of robot by varying the proportional and velocity gains during trajectory control loop and balancing loop. Stilman et al. [4] successfully developed and designed a novel humanoid torso named Golem Krang at Georgia Institute of Technology, Atlanta, Georgia, in the United States, which is capable of accessing the position in a constrained space in both static and dynamic mode. Golem krang resembles the dynamically stable two-wheel mobile robot attached with the manipulator.

Abeygunawardhana et al. [5] conducted simulation analysis and experiment to obtain high performance of stability in the two-wheel mobile manipulator. They succeeded to stabilize the posture of the two-wheel mobile manipulator by combining sliding mode controller using a twisting algorithm with disturbance observer. Ahmad et al. [6] established the mathematical dynamic model double-link two-wheeled mobile robot, which is a highly complex non-linear system. Hybrid fuzzy-PD type control system was implemented to control the links attached to the robot during dynamic stability simulation and proved the better performance prior to conventional PID controller. An et al. [7] developed mechanical structure and dynamic models by using energy conservation principle and implemented both Linear-quadratic regulator and proportional-integral-derivative control system for self-balancing and yaw rotation.

2. Dynamic system modelling

Here the dynamics of the robot is described by a mathematical model in order to facilitate the development of an efficient control system for balancing it. In this section, the linear model of a DC motor and equation of motion for a two-wheeled inverted pendulum is derived in detail. First of all a state space model for the DC motor is represented by considering the position and velocity as parameters. This model provides a relation between the input voltage to the motors and the control torque needed to balance the robot as shown in figure 3.

2.1. Linear Model of a DC Motor

Let V_a = applied voltage , i = current , K_m = torque constant, V_e = back e.m.f, τ_m = motor torque produced , K_e = back e.m.f constant, R = Resistance, I_R = initial load of armature, τ_a = applied torque , K_f = Frictional constant , ω = angular velocity of the shaft.

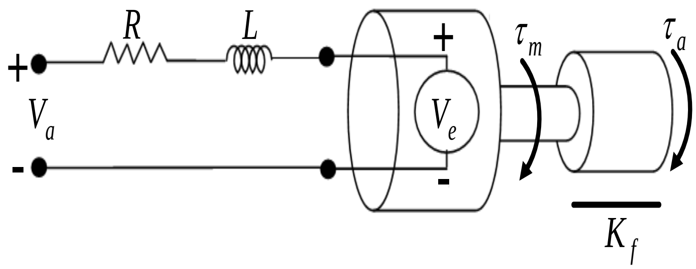

Figure 3 Linear model of a DC Motor

From basics of DC motor calculations we can write

$$V_a - V_e = iR \quad (1)$$

In addition, as V_e is proportional to rotation velocity of shaft and motor torque produced is proportional to current

$$V_e = K_e\omega \ , \ \tau_m = K_m i \quad (2)$$

$$V_a - K_e\omega = iR \quad (3)$$

$$i = \frac{-K_e\omega}{R} + \frac{1}{R}V_a \quad (4)$$

From the dynamics of motor and assuming motor inductance and motor friction to be negligible we can write

$$\tau_m = I_R\frac{d\omega}{dt} + \tau_a + K_f\omega \quad (5)$$

$$\frac{d\omega}{dt} = \frac{K_m i}{I_R} - \frac{\tau_a}{I_R} \quad (6)$$

From equation (4) and (6)

$$\frac{d\omega}{dt} = \frac{-K_e K_m\omega}{I_R R} + \frac{K_m}{I_R R}V_a - \frac{\tau_a}{I_R} \quad (7)$$

Hence, from the above equation the motor dynamics can be represented by following state space model.

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-K_m K_e}{I_R R} \end{bmatrix}\begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{K_m}{I_R R} & \frac{-1}{I_R} \end{bmatrix}\begin{bmatrix} V_a \\ \tau_a \end{bmatrix}$$

2.2. Dynamic Model of Two-Wheeled Cart

Let T_L, T_R = applied torque from left and right motors,

W_L, R_L, W_R, R_R = the reaction forces between wheels and chassis,

F_L, F_R = frictional force between ground and wheels, M_w = Mass of wheel, I_w = inertia of wheel, r = radius of wheel,

θ_w = Rotation of wheel as shown in figure 4, 5.

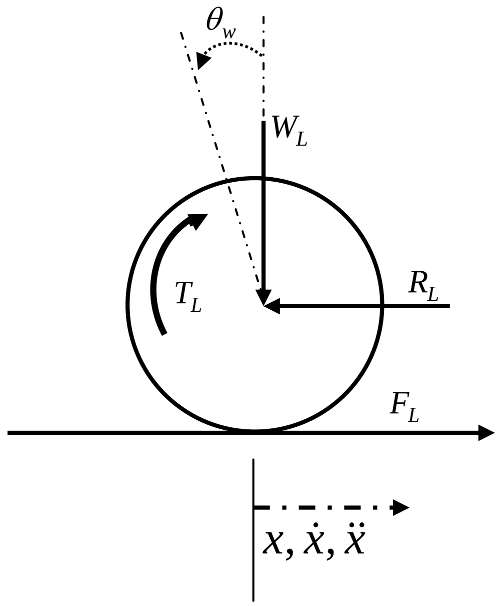

Figure 4 Free Body Diagram of wheel

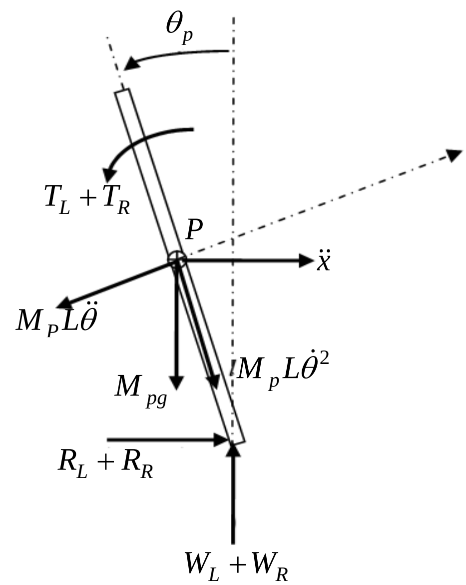

Figure 5 Free Body Diagram of Cart

For the wheel to be dynamically stable, we can write by applying Newton's law

$\Sigma F_x = M_a$ in x direction

$$M_w \frac{d^2x}{dt^2} = F_L - R_L \text{ (For left Wheel)}$$

And the moment about point 'O'

$$\Sigma M_o \Rightarrow I_w \frac{d^2\theta_w}{dt^2} = T_L - F_L r \quad (9)$$

Form DC motor dynamics

$$\tau_m = \frac{d\omega}{dt} I_A + \tau_a \quad (10)$$

$$\tau_M = T_L = \frac{-K_m K_e}{R}\left(\frac{d\theta w}{dt}\right) + \frac{K_m}{R} V_a \quad (11)$$

Putting in equation (2)

$$I_w\left(\frac{d^2\theta_w}{dt^2}\right) = -\frac{K_m K_e}{R}\left(\frac{d\theta_w}{dt}\right) + \frac{K_m V_a}{R} - F_L r \quad (12)$$

$$F_L = \frac{-I_w}{r}\left(\frac{d^2\theta_w}{dt^2}\right) - \frac{K_m K_e}{Rr}\left(\frac{d\theta_w}{dt}\right) + \frac{K_m V_a}{Rr} \quad (13)$$

Since linear and angular motion is acting on the centre of wheel

$$\frac{d^2\theta_w}{dt^2} = \frac{d^2x}{dt^2}\frac{1}{r} \text{ and } \frac{d\theta_w}{dt} = \frac{dx}{dt}\frac{1}{r} \quad (14)$$

$$F_L = -\frac{I_w}{r^2}\frac{d^2x}{dt^2} - \frac{K_m K_e}{Rr^2}\frac{dx}{dt} + \frac{K_m V_a}{Rr} \quad (15)$$

Similarly for right wheel

$$F_R = -\frac{I_w}{r^2}\frac{d^2x}{dt^2} - \frac{K_m K_e}{Rr^2}\frac{dx}{dt} + \frac{K_m V_a}{Rr} \quad (16)$$

Adding (15) and (16)

$$F_L + F_R = -\frac{2I_w}{r^2}\frac{d^2x}{dt^2} - \frac{2K_m K_e}{Rr^2}\frac{dx}{dt} + \frac{2K_m V_a}{Rr} \quad (17)$$

Adding both equation and substituting the value of $F_L + F_R$ from equation (17)

$$R_L + R_R = -2(M_w + \frac{I_w}{r^2})\frac{d^2x}{dt^2} - \frac{2K_m K_e}{Rr^2}\frac{dx}{dt} + \frac{2K_m V_a}{Rr} \quad (18)$$

Let M_p be mass of pendulum concentrated on point P of the pendulum at a distance *l*, θ_p is the angle tilt.

Using Newton's Law for dynamic stability

$$\Sigma F_x = M_p \frac{d^2x}{dt^2} \quad (19)$$

$$(R_L + R_R) - M_p l \frac{d^2\theta_p}{dt^2}\cos\theta_p + M_p l\left(\frac{d\theta_p}{dt}\right)^2 = M_p \frac{d^2x}{dt^2} \quad (20)$$

$$(R_L + R_r) = M_p \frac{d^2x}{dt^2} + M_p l \frac{d^2\theta_p}{dt^2}\cos\theta_p - M_p l\left(\frac{d\theta_p}{dt}\right)^2 \sin\theta_p \quad (21)$$

$$\Sigma T_p = I_p \frac{d^2\theta_p}{dt^2} \quad (22)$$

$$(W_L + W_R) l \sin\theta_p - (R_L + R_R) l \cos\theta_p - (T_L + T_R) = I_p \frac{d^2\theta_p}{dt^2} \quad (23)$$

Since $(T_L + T_R) = \left[\frac{-K_m K_e}{Rr}\frac{dx}{dt} + \frac{K_m}{R}V_a\right] \times 2$ i.e the torque applied on pendulum from motor. (24)

The sums of forces perpendicular to the pendulum are

$$\Sigma F_{xp} = M_p \ddot{x}\cos\theta_p \quad (25)$$

$$(R_L + R_R)\cos\theta_p + (W_L + W_R)\sin\theta_p - M_p g \sin\theta_p - M_p l\ddot{\theta}_p = M_p \ddot{x}\cos\theta_p \quad (26)$$

Multiplying (26) by $-l$ and adding with equation (23)

$$-(T_L + T_R) = M_p g l \sin\theta_p + M_p l \frac{d^2\theta_p}{dt^2} + I_p \frac{d^2\theta_p}{dt^2} + M_p \frac{d^2x}{dt^2}\cos\theta_p \quad (27)$$

From (24)

$$\frac{2K_m K_e}{Rr}\frac{dx}{dt} - \frac{2K_m V_a}{R} = M_p g l \sin\theta_p + M_p l \frac{d^2\theta_p}{dt^2} + I_p \frac{d^2\theta_p}{dt^2} + M_p \frac{d^2x}{dt^2}\cos\theta_p \quad (28)$$

$$(I_p + M_p l^2)\frac{d^2\theta_p}{dt^2} - \frac{2K_m K_e}{Rr}\left(\frac{dx}{dt}\right) + \frac{2K_m V_a}{R} + M_p g l \sin\theta_p = -M_p l \frac{d^2x}{dt^2}\cos\theta_p \quad (29)$$

$$\frac{2K_m V_a}{Rr} = \left(2M_w + \frac{2I_w}{r^2} + M_p\right)\frac{d^2x}{dt^2} + \frac{2K_m K_e}{Rr^2}\frac{dx}{dt} + M_p l \frac{d^2\theta_p}{dt^2}\cos\theta_p - M_p l \dot{\theta}_p{}^2 \sin\theta_p \quad (30)$$

The two non-linear equations of motion of the system can be rearranged as,

$$(I_p + M_p l^2)\ddot{\theta}_p - \frac{2K_m K_e}{Rr}\dot{x} + \frac{2K_m}{R}V_a + M_p g l \sin\theta_p = M_p l \cos\theta_p \quad (31)$$

$$\frac{2K_m}{Rr}V_a = \left(2M_w + \frac{2I_w}{r^2} + M_p\right)\ddot{x} + \frac{2K_m K_e}{Rr^2}\dot{x} + M_p l \ddot{\theta}_p \cos\theta_p - M_p l \dot{\theta}_p^2 \sin\theta_p \quad (32)$$

The above two equations can be linearised by assuming $\theta_p = \pi + \phi$

ϕ is very small angle, $\cos\theta_p = -1, \sin\theta_p = -\phi$, $\left(\frac{d\theta_p}{dt}\right)^2 = 0$

The linearised equation of motion is

$$\left(I_p + M_p l^2\right)\ddot{\phi} - \frac{2K_m K_e}{Rr}\dot{x} + \frac{2K_m}{R}V_a - M_p g l\phi = M_p l\ddot{x} \quad (33)$$

$$\frac{2K_m}{Rr}V_a = \left(2M_w + \frac{2I_w}{r^2} + M_p\right)\ddot{x} + \frac{2K_m K_e}{Rr^2}\dot{x} - M_p l\ddot{\phi} \quad (34)$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{2K_m K_e\left(M_p lr - I_p - M_p l^2\right)}{Rr^2\alpha} & \frac{M_p^2 g l^2}{\alpha} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2K_m K_e\left(r\beta - M_p l\right)}{Rr^2\alpha} & \frac{M_p g l\beta}{\alpha} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2K_m\left(I_p + M_p l^2 - M_p lr\right)}{Rr\alpha} \\ 0 \\ \frac{2K_m\left(M_p l - r\beta\right)}{Rr\alpha} \end{bmatrix} V_a \quad (35)$$

Where $\beta = \left[2M_w + \frac{2I_w}{r^2} + M_p\right]$, $\alpha = \left[I_p\beta + 2M_p L^2\left(M_w + \frac{I_w}{r^2}\right)\right]$

3. DISCUSSIONS

In the above analysis, the linear state space representation of angular velocity and angular acceleration of DC motor model has been established considering various parameters like motor torque, applied voltage, inertial load on armature and motor resistance. Similarly a dynamic model for a two wheeled manipulator is being formulated considering both force and moment equations on wheel as well as cart. This corresponds to final state space equation representing cart's linear along with angular velocity and acceleration.

4. CONCLUSIONS

The two-wheeled mobile manipulators show an excellent maneuverability in confined space with smaller footprints and more flexibility than statically stable multi wheel counter parts. These are differential drive robots consisting of two wheels on a common axis and each wheel driving independently. Such arrangement gives the robot the ability to drive straight, turn in narrow place and move in arc. But sometimes the errors coming from the wheel site, friction between the wheels and contact surface deviate the robot from its intended course. Due to its nonlinear and unstable system dynamics it is really a challenging task to design a suitable control system for a two wheeled balancing robot. As the complete state of the system cannot be fully measured often, hence it is essential to derive the linear state space controllers utilizing the dynamic model. Hence in this current research is emphasized on mathematical establishment of dynamic system modelling through space representation. Also as the system is inherently unstable, suitable design of some efficient controller for stabilizing the robot is taken as future aspect.

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